APPROXIMATE METHOD OF CONVERTING THE LAW OF ONE-DIMENSIONAL SHOCK PROPAGATION

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A method is proposed for the approximate calculation of the correction to the universal law of one-dimensional shock propagation in a polytropic gas at rest in the case when the wave proceeds over an inhomogeneous medium.

The laws of one-dimensional explosive shock propagation in a polytropic gas at rest with constant pressure p_{01} and density ρ_{01} possess the property of universality in the sense that the relative pressure jump $q_0=(p_{02}-p_{01})/p_{01}$ in such a shock, considered as a function of the relative distance $\xi=r/r_0$ (for a suitable selection of the length scale r_0), is independent of the quantity p_{01}. These laws have been studied well enough. In particular, detailed computed tables [1] and approximation formulas [2] exist for a point explosion.

If the initial pressure distribution in the gas $p_1=p_1(\xi)$ is not constant while remaining one-dimensional, then the relative pressure jump in the shock $q=(p_2-p_1)/p_1$ will already not be a universal function of ξ but will depend on the whole distribution $p_1(\xi')$ for $\xi'<\xi$. Consequently, it is necessary to write the appropriate dependence as $q=q(\xi, p_1)$. An analytical investigation of the operator dependence of q on $p_1(\xi)$ is a quite complex problem to whose solution no direct approach is as yet visible. Hence, proposals for an approximate analysis of this dependence, which should result in methods of converting the universal law $q_0(\xi)$ into the law $q(\xi, p_1)$, are meaningful. This paper is indeed devoted to the construction of one such approximation.

Both dependences ($q_0(\xi)$ and $q(\xi, p_i)$) are compared at those points at which the quantities q_0 and q are equal. This results in the problem of seeking a function $\eta(\xi)$ for which there will be compliance with the equality

$$q(\xi,\ p_1) = q_0(\eta(\xi)) \tag{1}$$

At some point ξ and $\eta=\eta(\xi)$ let (1) be satisfied. It is continued to the point $\xi+d\xi$, $d\xi>0$ by comparing differentials of the left and right sides. For the right side this will simply be

$$dq_0 = q_0'(\eta)\frac{d\eta}{d\xi}d\xi \tag{2}$$

where the prime will denote the derivative of the function with respect to the denoted argument here and everywhere henceforth.

The differential of the left side is represented as

$$dq(\xi,\ p_1) = \partial q + \delta q \tag{3}$$

where ∂q is taken for the distribution coinciding with the given more leftward point ξ and equal to $p_1(\xi)$ for the more rightward point ξ, and δq denotes the differential taken in conformity with the change in $p_1(\xi)$ from the value p_1 to p_1+s, where $s=p_1'(\xi)\,d\xi$. Two simplifying assumptions are made for the approximate calculation of these differentials.

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1°. The quantity ∂q is independent of the "history" of the motion. This means that if the changed pressure p_1 remained constant for $d\xi > 0$ and equal to $p_1(\xi)$, then for $d\xi > 0$ the shock will be propagated according to the universal law. Since (1) is satisfied at the point under consideration, it then follows from Proposition 1° that the "rate of change" of the quantity q in ∂q equals $q_0'(\eta)$, whereupon

$$\partial q = q_0'(\eta)\, d\xi \tag{4}$$

2°. The quantity δq is formed because the shock at the point ξ meets the gas at rest with the pressure $p_{1S} = p_1 + s$, and then dissociation occurs of the discontinuity between the state 2 of the shock arriving at the point ξ and the state 1s. Let a shock with pressure p_{2S} on the front behind the shock be formed in this dissociation of the discontinuity. Then the changed value of q will equal $q_S = (p_{2S} - p_{1S})$. Proposition 2° reduces to the fact that δq is assumed to equal the principal part of the increment $q_S - q$. Hence, taking account of only a small first order quantity in s yields

$$\delta q = \frac{\partial}{\partial s}\left(\frac{p_{2s}}{p_{1s}}\right)\Bigg|_{s=0} p_1'(\xi)\, d\xi \tag{5}$$

It is convenient to introduce a function defined completely by the above-mentioned dissociation of the discontinuity,

$$\psi(q) = p_1 \frac{\partial}{\partial s}\left(\frac{p_{2s}}{p_{1s}}\right)\Bigg|_{s=0} \tag{6}$$

Taking account of (4), (5) and the notation (6) in a comparison of (2) with (3) results, after division by $q_0'(\eta)\, d\xi$, in the relationship

$$\frac{d\eta}{d\xi} = 1 + \frac{\psi(q_0(\eta))}{q_0'(\eta)} \frac{d}{d\xi} \ln p_1(\xi) \tag{7}$$

Since the right side in (7) is a known function of the variables ξ, η, then (7) is the differential equation to determine the desired dependence $\eta(\xi)$. After substitution into (1), its solution under the initial condition $\eta(0) = 0$ will yield the relative pressure jump in the shock traveling over the distribution $p_1(\xi)$.

Thus, the method proposed for the conversion reduces to the integration of (7) and the use of (1).

There remains to evaluate the function $\psi(q)$ by means of (6). To do this, the equation for the dissociation of the discontinuity between the left state 2 and the right state 1s in a polytropic gas with adiabatic index γ (u_2 is the mass flow rate in the left state) is used

$$u_2 = \sqrt{2\tau_2}\, \frac{p_{2s} - p_2}{\sqrt{(\gamma+1)\, p_{2s} + (\gamma-1)\, p_2}} + \sqrt{2\tau_{1s}}\, \frac{p_{2s} - p_{1s}}{\sqrt{(\gamma+1)\, p_{2s} + (\gamma-1)\, p_{1s}}} \tag{8}$$

where the specific volumes τ are given by the formulas

$$\tau_2 = \tau_1 \frac{(\gamma-1)\, p_2 + (\gamma+1)\, p_1}{(\gamma+1)\, p_2 + (\gamma-1)\, p_1}, \qquad \tau_{1s} = \tau_1 \left(\frac{p_{1s}}{p_1}\right)^{1/\gamma}$$

Differentiating (8) with respect to s at the point $s = 0$ and taking into account that $p_2 = (1+q)\, p_1$ yields an equation for ψ from which the following result is obtained:

$$\psi(q) = -\frac{2(1+\mu q)\sqrt{1+q}\,\left[\sqrt{1+(1-\mu)\, q}\,\sqrt{1+q} + (1-\mu)\, q\right]}{2(1+\mu q)\sqrt{1+(1-\mu)\, q} + (2+\mu q)\sqrt{1+q}}$$
$$\mu = \frac{(\gamma+1)}{2\gamma} \tag{9}$$

For the case of small values of q which is important in practice, the approximate representation resulting from (9) can be used,

$$\psi(q) \approx -{}^1/_2\, [1 + (2-\mu)\, q]$$

In conclusion, the author considers it his duty to note that the idea for realization of such an approximate method of conversion was proposed by V. A. Bronshten and that this paper evolved as a result of joint fruitful discussions.

LITERATURE CITED

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